Mechanics of composites

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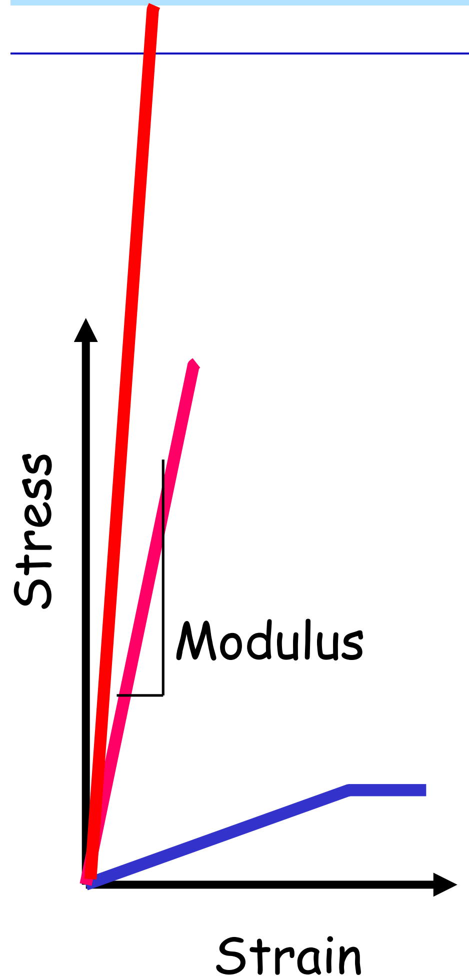
https://altairhyperworks.com/product/ESAComp

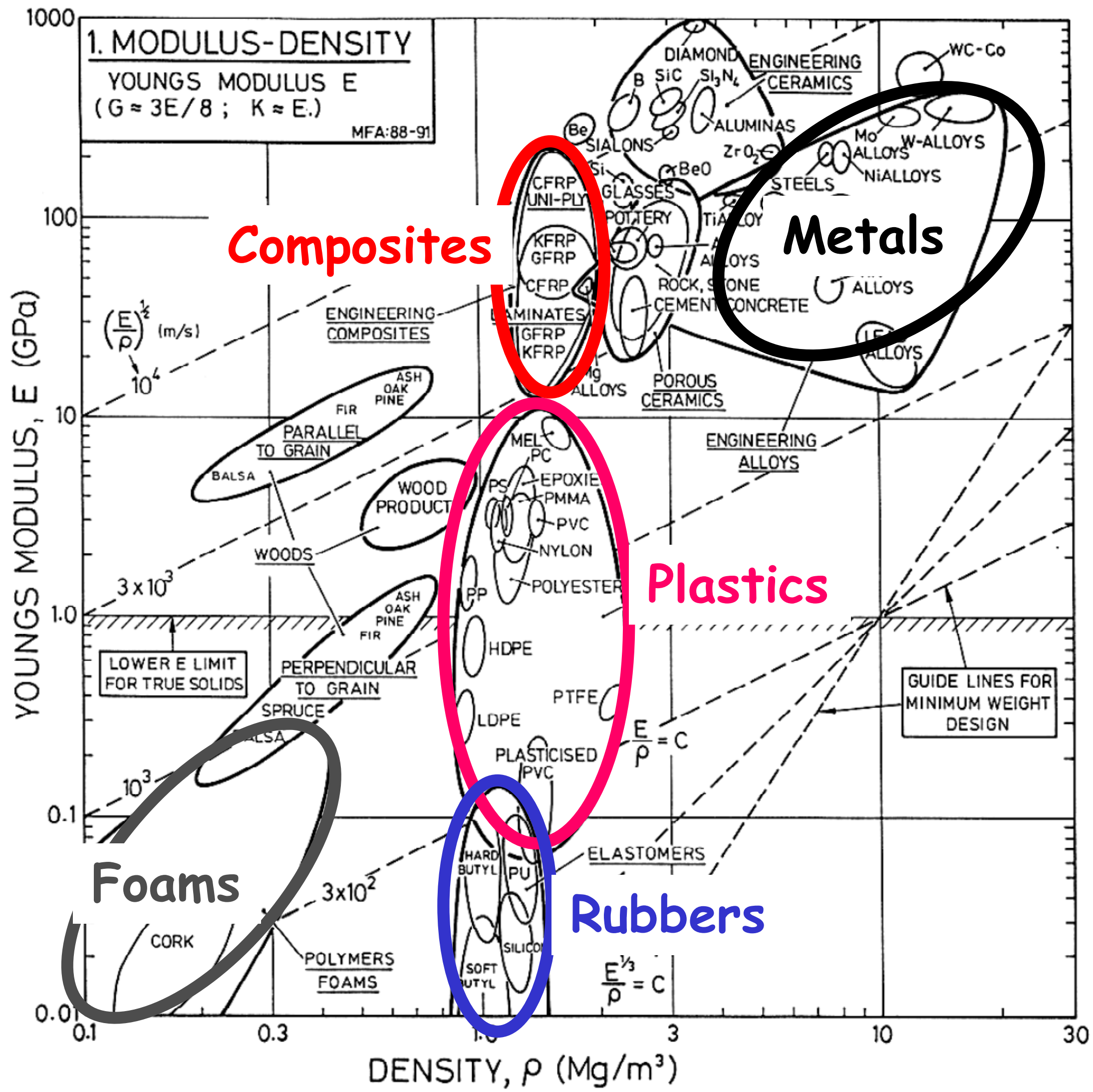
CoDA ou LAP sur http://www.anaglyph.co.uk

http://www.efunda.com/formulae/solid_mechanics/composites/calc_ufrp_abd_layout.cfm

WiseTex sur http://wisetexdemo.software.informer.com/

Siffness versus density





M.F. Ashby, Materials Selection for Mechanical design, Pergamon Press

Freedom in reinforcement types

Materials

Glass — Aramid — Steel — Carbon

0 — *E Modulus (Gpa)* — *400*

Shapes

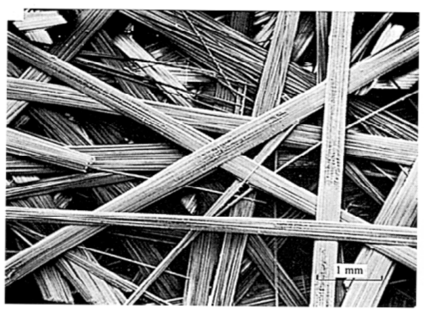
Particulates — Fibres: short — long — discontinuous — continuous

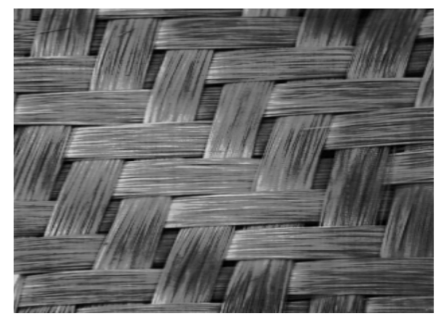
1 — *Shape factors L/d* — *∞*

Architectures

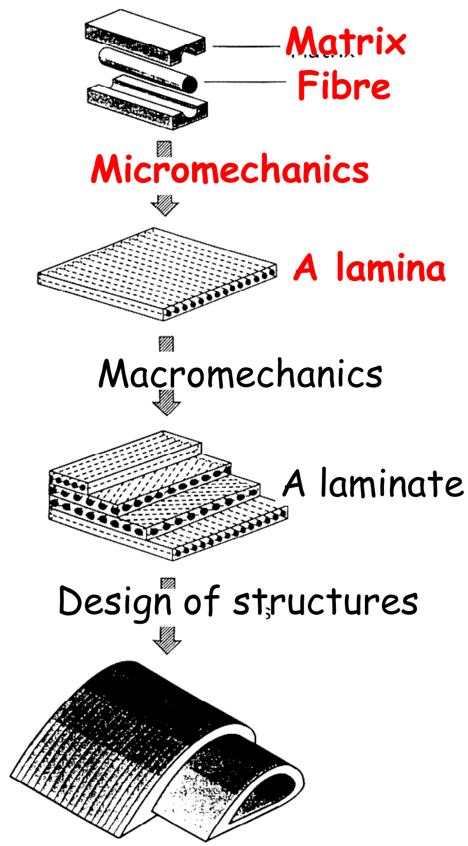
Random — Knitts — Weaves

0 — *Orientation* — *1*

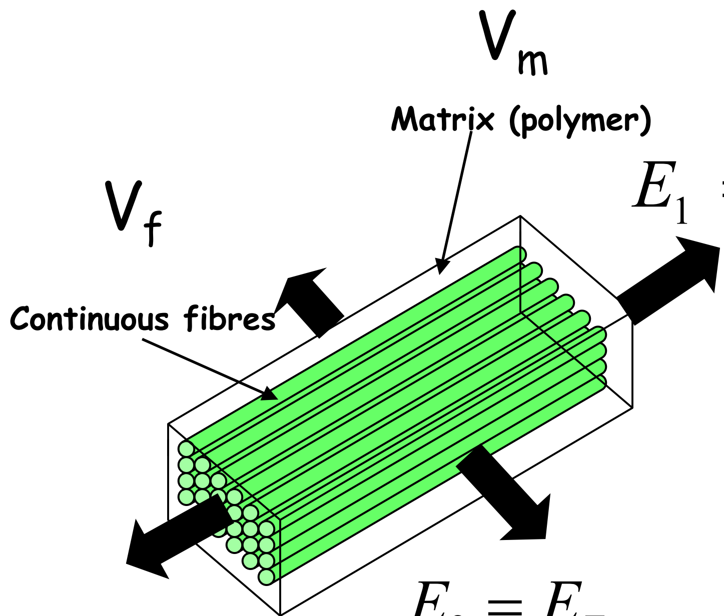




Micromechanics



Continuous fibres



$$E_1 = E_{Longitudinal} = E_f V_f + E_m (1 - V_f)$$

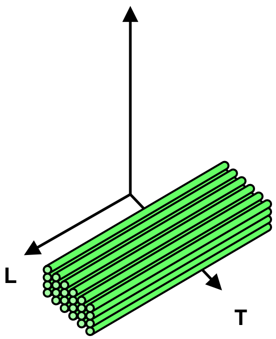
$$E_2 = E_{Transverse} = \frac{E_m E_f}{E_f (1 - V_f) + E_m V_f}$$

$$\nu_{LT} = V_f \nu_f + V_m \nu_m$$

$$\alpha_L = \frac{\alpha_f E_f V_f + \alpha_m E_m V_m}{E_f V_f + E_m V_m}$$

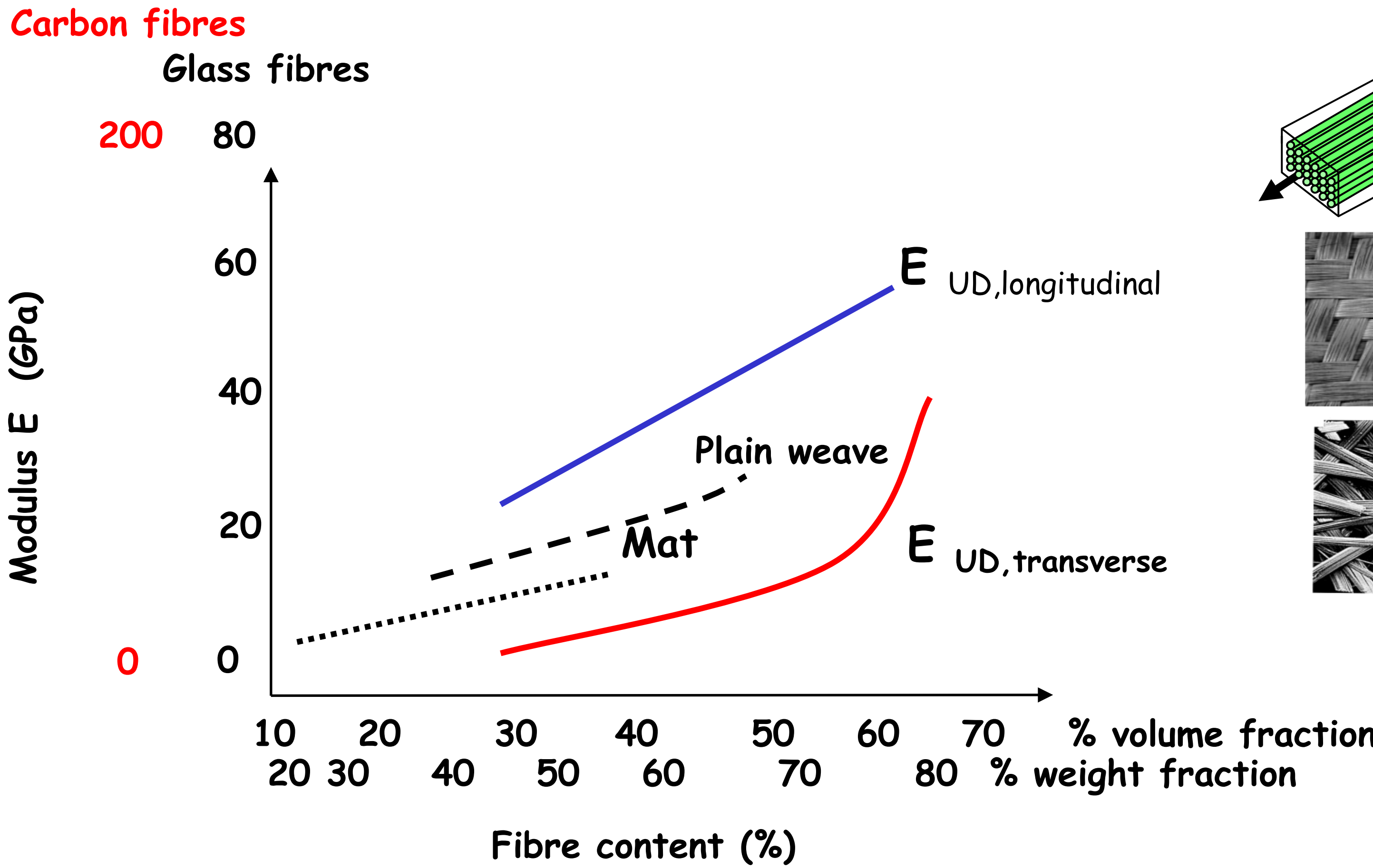
$$\alpha_T = (1 + \nu_m)\alpha_m V_m + (1 + \nu_f)\alpha_f V_f - \alpha_L \nu_{LT}$$

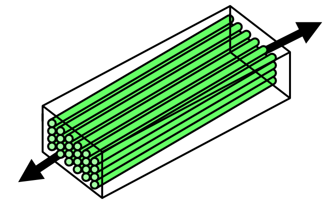
Properties of unidirectional composites

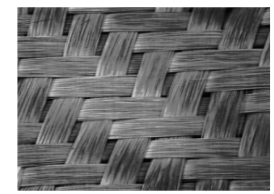


	S Glass Epoxy	Kevlar 49 Epoxy	Carbon HR Epoxy	Carbon HM Epoxy	Boron Epoxy
Vf	65 %	65 %	65 %	65 %	55 %
Density ρ	2.04 g/cm^3	1.36 g/cm^3	1.56 g/cm^3	1.5 g/cm^3	1.97 g/cm^3
E_L	56 GPa	86 GPa	145 GPa	270 GPa	220 GPa
E_T	16 GPa	5.6 GPa	10 GPa	7 GPa	2.3 GPa
ν_{LT}	0.26	0.32	0.29	0.3	0.26
G_{LT}	7 GPa	2.5 GPa	5.5 GPa	5.7 GPa	6.9 GPa
σ_{rL} **tension**	1.75 GPa	1.5 GPa	1.2 GPa	0.95 GPa	1.3 GPa
σ_{rT} **tension**	0.04 GPa	0.03 GPa	0.08 GPa	0.035 GPa	0.065 GPa
σ_{rL} **compression**	0.9 GPa	0.28 GPa	1 GPa	0.75 GPa	2.85 GPa
σ_{rT} **compression**	0.15 GPa	0.14 GPa	0.25 GPa	0.2 GPa	0.03 GPa
γ_{rLT} **shear**	0.06 GPa	0.05 GPa	0.1 GPa	0.055 GPa	0.06 GPa

Which stiffness do you need ?

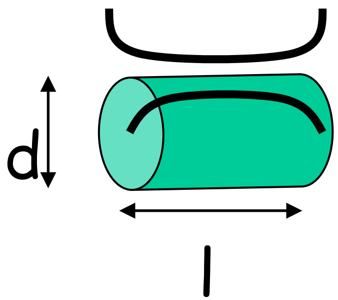






Short fibre composites

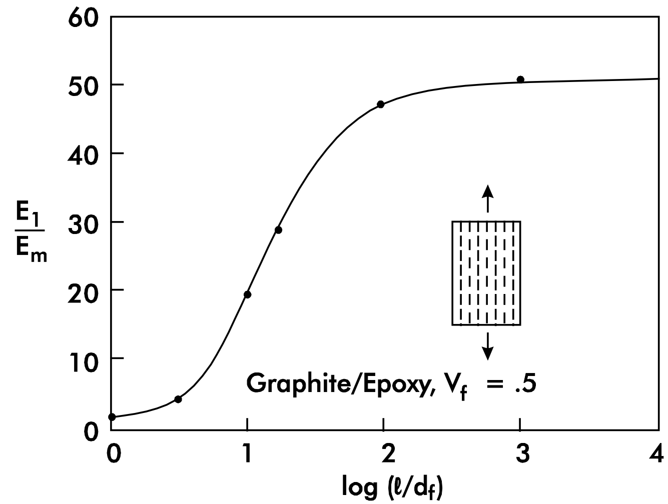
$$E = \eta_{\text{orientation}} \eta_{\text{fiber length}} E_f V_f + \text{E}_{\text{m}} (1 - V_f)$$



$$\text{l}_{\text{c}} = \frac{\text{d}\ \sigma_{\text{f, ult}}}{2\ \tau_{\text{y}}}$$

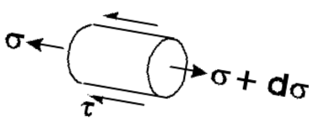
Short fibre composites

Length of fibres



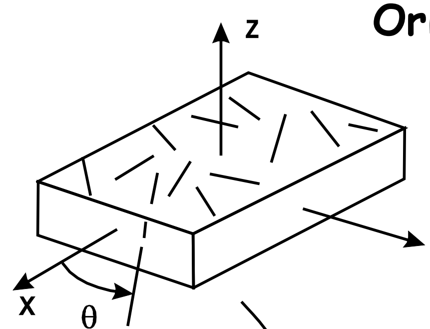
Cox model

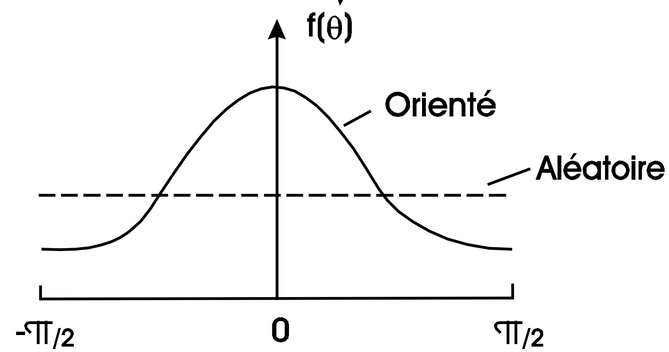
$$E_I = E_f V_f \left[1 - \frac{\tanh(\beta \ell/2)}{\beta \ell/2} \right] + E_m V_m$$



$$\tau = c(u_f - u_m) \qquad \beta^2 = \frac{4c}{d_f E_f}$$

Orientation of fibres





$$f(\theta) = \left(\frac{1}{\pi}\right) \frac{1 - \lambda^2}{1 + \lambda^2 - 2\lambda\cos(2\theta)}$$

where λ is the 'orientation parameter'. For random distribution, λ=0, for more oriented fibres , λ takes higher values, 0<λ<1.

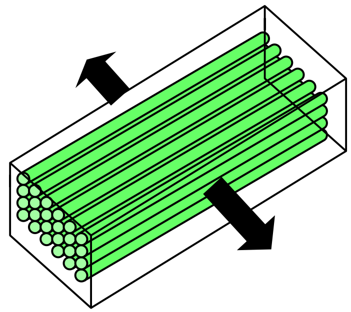
LPAC Laboratory for Processing of Advanced Composites

EPFL

Halpin-Tsai equations

$$P = \frac{P_m(1+\xi\,\chi\,V_f)}{1-\chi\,V_f}$$

$$\chi = \frac{P_f - P_m}{P_f + \xi\,\mathrm{P_m}}$$

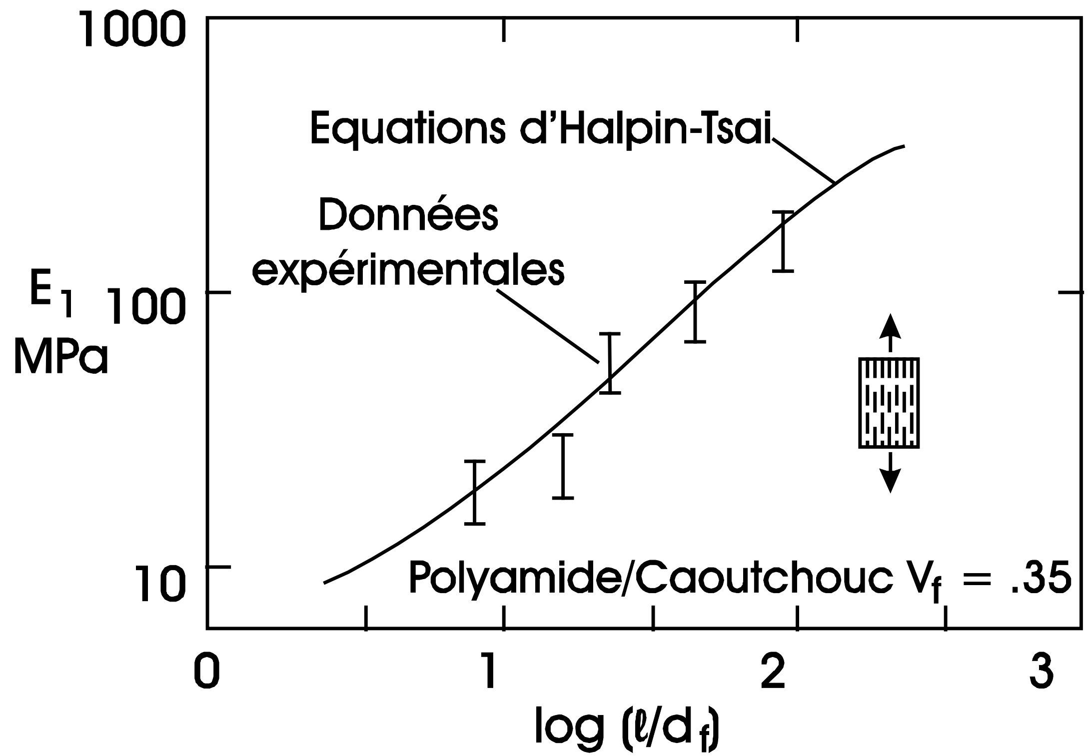


$$\xi(\mathrm{E_T}) = 2$$

$$\xi(\mathrm{G_{LT}}) = 1$$

$$\xi \approx \frac{2\,l}{\mathrm{d}}$$

Short fibre composites



Properties of short fibre composites

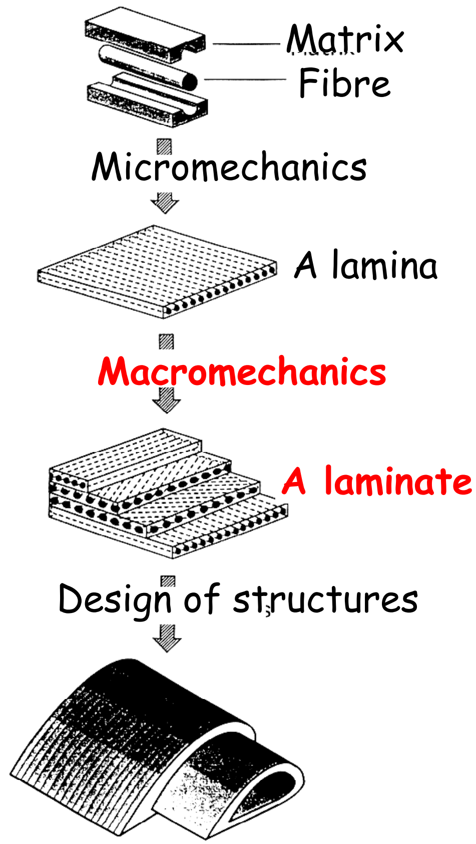
Properties at 23°C	Zytel®		Zytel® 40% short fibres		Zytel® 50% short fibres	
	0 % HR	50% HR	0 % HR	50% HR	0 % HR	50% HR
Yiel stress σ_y (MPa)	84 MPa	48 MPa	205 MPa	135 MPa	230 MPa	155 MPa
Failure strain ε_y (%)	50 %	>300 %	3 %	6 %	2 %	5 %
Bending modulus E	2.7 GPa	0.9 GPa	10.5 GPa	6.5 GPa	23.5 GPa	8.5 GPa
Shock (notched) Izod	50 J/m	200 J/m	160 J/m	214 J/m	180 J/m	270 J/m
Shock resistance Charpy	Pas de rupture		60 kJ/m^2		65 kJ/m^2	
Density ρ	1.14 g/cm^3		1.45 g/cm^3		1.58 g/cm^3	
Melting temperature	245°C		233 °C		233 °C	
Heat distortion temperature in bending at 1.8 MPa	65 °C		224 °C			
Water absorption 24h (immersion)	1.6 %					
Molding shrinkage	1.3 %		0.18 %		0.16 %	

matweb.com

Other models

- with anisotropic fibres
- for 2D and 3D composites

Macromechanics



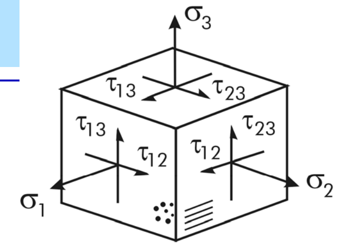
Introduction

$$\sigma_{ij} = f(\varepsilon_{kl})$$

$$\begin{bmatrix}\sigma_1\\\sigma_2\\\sigma_3\\\sigma_4\\\sigma_5\\\sigma_6\end{bmatrix} = \begin{bmatrix}C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36}\\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46}\\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56}\\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_3\\\varepsilon_4\\\varepsilon_5\\\varepsilon_6\end{bmatrix}$$

$$[C] = \begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}$$

Anisotropic materials



<u>81 csts</u>

Linear elasticity

Hooke'slaw

Stresses and strains are symmetric <u>36 csts</u>

Tensor is symmetric <u>21 csts</u>

Material symmetry

- Monoclinical <u>13 csts</u>
- Orthotropy <u>9 csts</u>
- Transverse isotropy <u>5 csts</u>
- Isotropy <u>2 csts</u>

Approach used by the engineers

Constants of the engineer

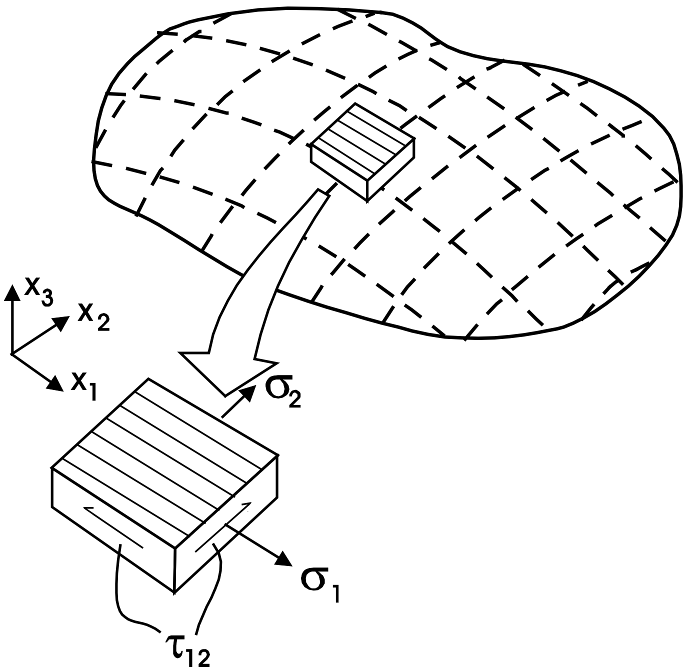
Orthotropic materials under plane stresses

Laminate theory and effective properties

Laminate symmetries and coupling effects

Design maps

Orthotropic materials under plane stresses



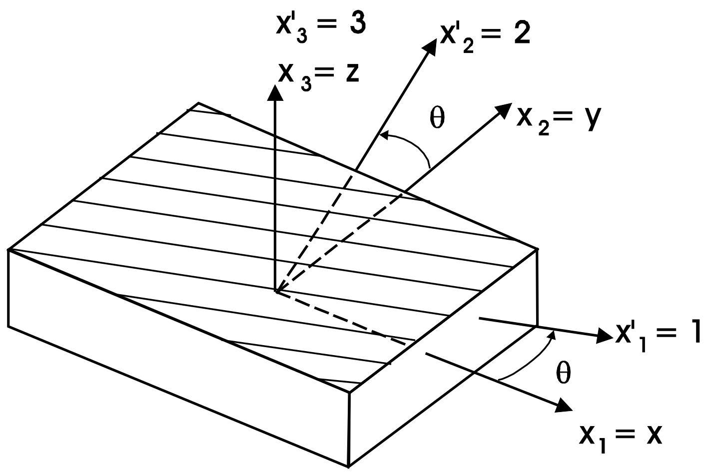
$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix}$$

$$Q_{11}=\frac{E_1}{(1-\nu_{12}\nu_{21})} \qquad Q_{22}=\frac{E_2}{(1-\nu_{12}\nu_{21})}$$

$$Q_{12}=\frac{\nu_{12}E_2}{(1-\nu_{12}\nu_{21})}=\frac{\nu_{21}E_1}{(1-\nu_{12}\nu_{21})} \qquad Q_{66}=G_{12}$$

$$\left[S_{ij}\right]=\begin{bmatrix}S_{11} & S_{12} & 0\\ S_{12} & S_{22} & 0\\ 0 & 0 & S_{66}\end{bmatrix}=\begin{bmatrix}1/E_1 & -\upsilon_{21}/E_2 & 0\\ -\upsilon_{12}/E_1 & 1/E_2 & 0\\ 0 & 0 & 1/G_{12}\end{bmatrix}$$

Importance of fibre orientation



m=cos, n=sin

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}$$

$$\overline{Q}_{11}=m^4Q_{11}+2m^2n^2(Q_{12}+2Q_{66})+n^4Q_{22}$$

$$\overline{Q}_{21}=\overline{Q}_{12}=m^2n^2(Q_{11}+Q_{22}-4Q_{66})+Q_{12}(m^4+n^4)$$

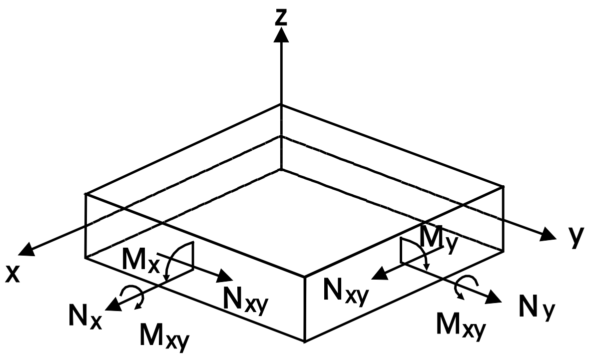
$$\overline{Q}_{22}=n^4Q_{11}+2m^2n^2(Q_{12}+2Q_{66})+m^4Q_{22}$$

$$\overline{Q}_{16}=m^3n(Q_{11}-Q_{12})+mn^3(Q_{12}-Q_{22})-2mn(m^2-n^2)Q_{66}$$

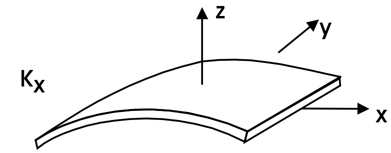
$$\overline{Q}_{26}=mn^3(Q_{11}-Q_{12})+m^3n(Q_{12}-Q_{22})+2mn(m^2-n^2)Q_{66}$$

$$\overline{Q}_{66}=m^2n^2(Q_{11}+Q_{22}-2Q_{12}-2Q_{66})+(m^4+n^4)Q_{66}$$

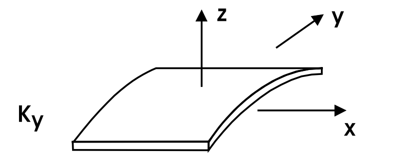
Laminate elasticity



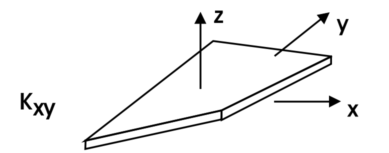
$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix}$$



$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

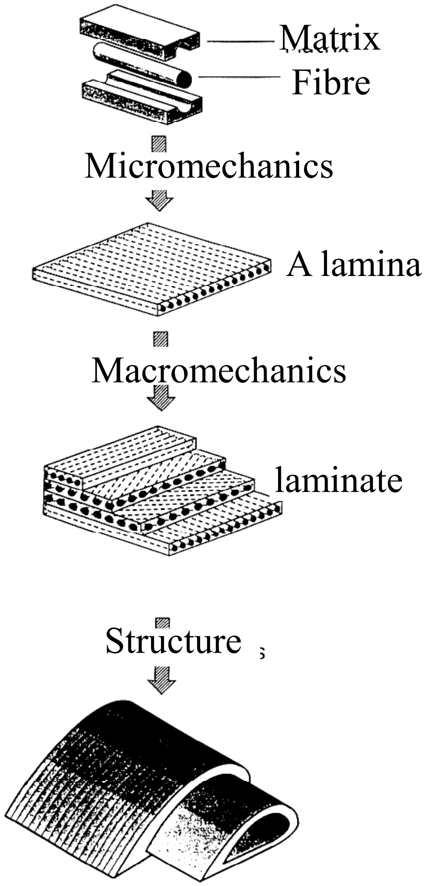


$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$



$$A_{ij} = \sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k \left(z_k - z_{k-1}\right) \qquad B_{ij} = \frac{1}{2} \sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k \left(z_k^2 - z_{k-1}^2\right) \qquad D_{ij} = \frac{1}{3} \sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k \left(z_k^3 - z_{k-1}^3\right)$$

From fibre to structures



$$E_1 = E_f V_f + E_m (1 - V_f) \qquad P = \frac{P_m (1 + \xi \chi V_f)}{1 - \chi V_f}$$

$$Q_{11} = \frac{E_1}{(1 - \nu_{12}\nu_{21})}$$

$$\overline{Q}_{11} = m^4 Q_{11} + 2m^2 n^2 (Q_{12} + 2Q_{66}) + n^4 Q_{22}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \qquad \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$A_{ij} = \sum_{k=1}^{N} (\overline{Q}_{ij})_k (z_k - z_{k-1})$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & a_{66} \end{bmatrix} \left\{ \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} + \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} \right\}$$

$$E_x = \frac{A_{11} A_{22} - A_{12}^2}{h A_{22}}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 \\ -\frac{\nu_{yx}}{E_y} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

Elastic materials
No porosity
Interfaces with good adhesion of matrix to fibres

Fibes orientations

Orthotropy
Plane stress

Good adhesion bewteen plies

Symmetry in stacking sequence

Coupling effects

Load case
Effective properties
Mechanics of materials

Symmetric and balanced laminates

Laminate	A_{11} (MN/m)	A_{22} (MN/m)	G_{xy} (GPa)
$[90_8]_s$	11.09	153.28	2.29
$[0/90_7]_s$	28.86	135.51	2.29
$[0_2/90_6]_s$	46.64	117.73	2.29
$[0_3/90_5]_s$	64.41	99.96	2.29
$[0_4/90_4]_s$	82.18	82.18	2.29
$[0_5/90_3]_s$	99.96	64.41	2.29
$[0_6/90_2]_s$	117.73	46.64	2.29
$[0_7/90]_s$	135.51	28.86	2.29
$[0_8]_s$	153.28	11.09	2.29
$[\pm 45/90_6]_s$	20.21	126.85	6.62
$[\pm 45/0/90_5]_s$	37.98	109.08	6.62
$[\pm 45/0_2/90_4]_s$	55.76	91.31	6.62
$[\pm 45/0_3/90_3]_s$	73.53	73.53	6.62
$[\pm 45/0_4/90_2]_s$	91.31	55.76	6.62
$[\pm 45/0_5/90]_s$	109.08	37.98	6.62
$[\pm 45/0_6]_s$	126.85	20.21	6.62
$[\pm 45_2/90_4]_s$	29.33	100.43	10.95
$[\pm 45_2/0/90_3]_s$	47.11	82.65	10.95
$[\pm 45_2/0_2/90_2]_s$	64.88	64.88	10.95
$[\pm 45_2/0_3/90]_s$	82.65	47.11	10.95
$[\pm 45_2/0_4]_s$	100.43	29.33	10.95
$[\pm 45_3/90_2]_s$	38.45	74.00	15.27 F
$[\pm 45_3/0/90]_s$	56.23	56.23	15.27 F
$[\pm 45_3/0_2]_s$	74.00	38.45	15.28 F
$[\pm 45_4]_s$	47.58	47.58	19.60 F

Effective properties

Symmetric and balanced

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} + \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & a_{66} \end{bmatrix} \left\{ \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} + \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} \right\}$$

Only mechanical loading
Uniform strain through the thickness

$$\varepsilon_x^0 = a_{11} N_x = \varepsilon_x$$

$$\sigma_x = \frac{N_x}{h}$$

$$\varepsilon_x = a_{11}\, h\, \sigma_x$$

$$E_x = \frac{\sigma_x}{\varepsilon_x} = \frac{1}{h\, a_{11}}$$

$$E_x = \frac{A_{11}A_{22} - A_{12}^2}{h\, A_{22}}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 \\ -\frac{\nu_{yx}}{E_y} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

Effective engineering properties

to be used in material mechanics equations

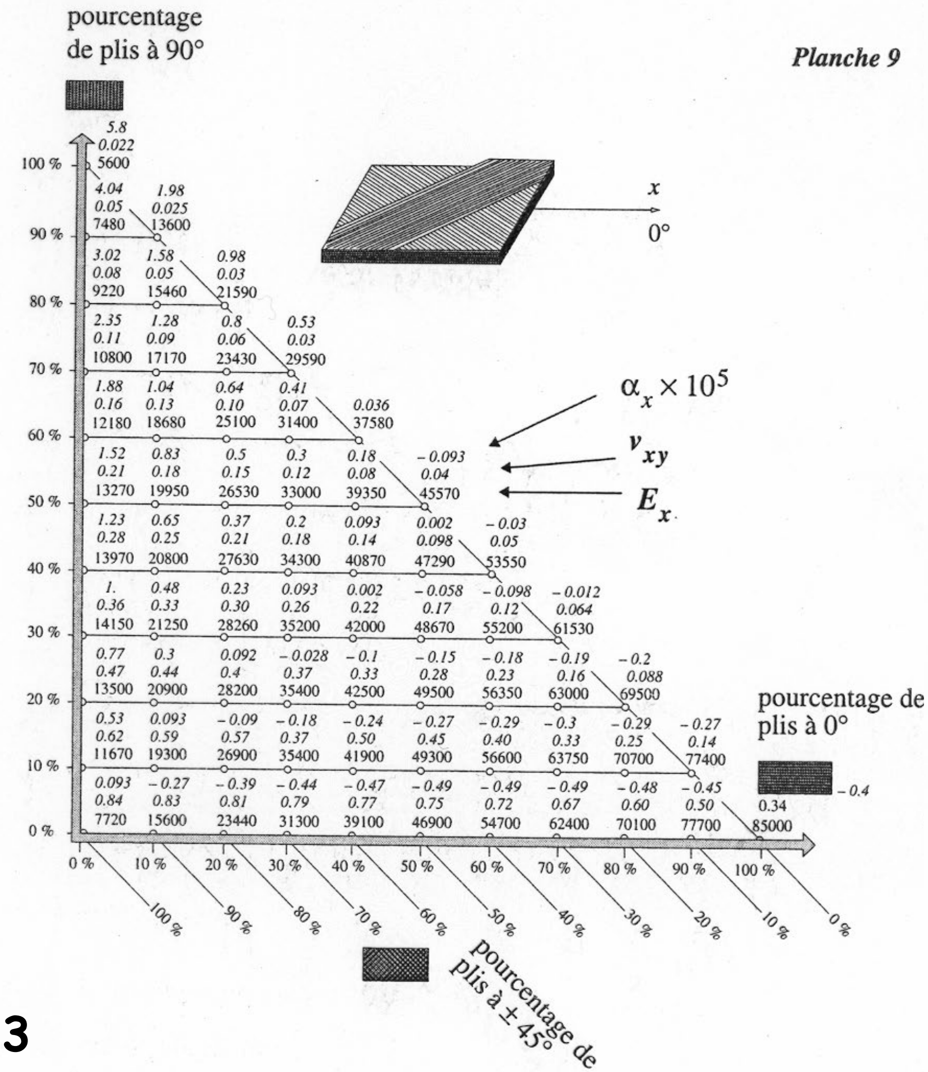
Stratifié kevlar/époxyde

$V_f = 60\ \%$ de fibres en volume

épaisseur du pli : 0.13 mm

caractéristiques du pli : *cf.* paragraphe 3.3.3.

Planche 9



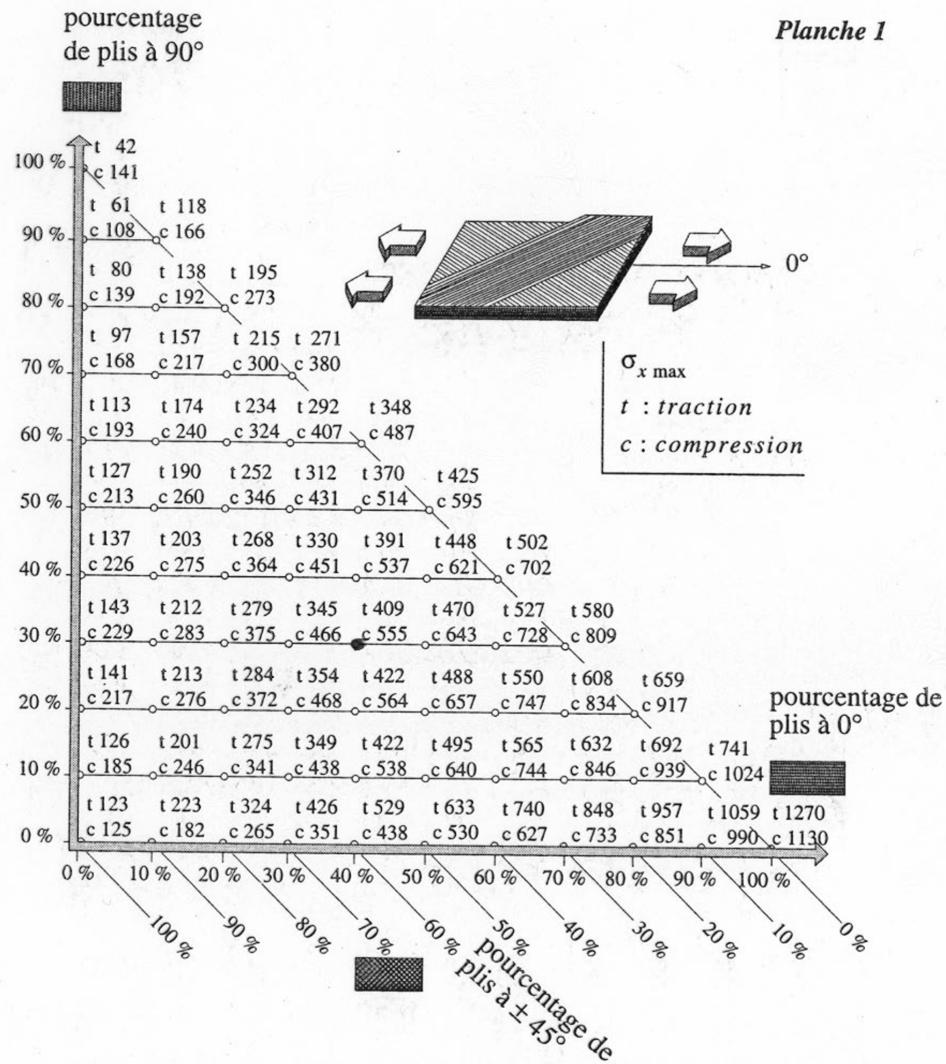
Stratifié carbone/époxyde

$V_f = 60\ \%$ de fibres en volume

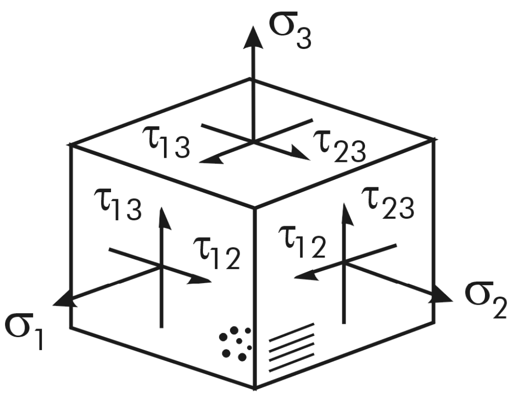
épaisseur du pli : 0.13 mm

caractéristiques du pli : modules, contraintes de rupture : *cf.* paragraphe 3.3.3.

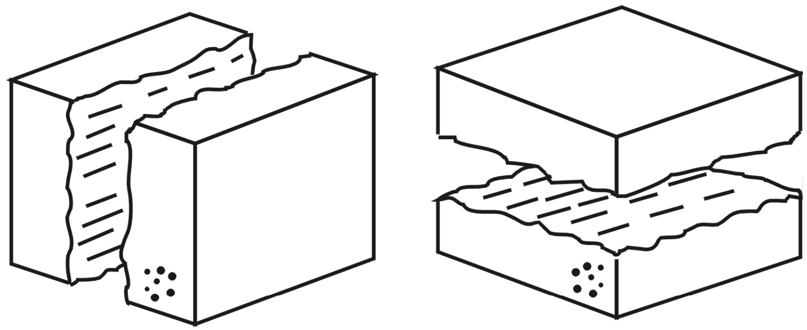
Planche 1



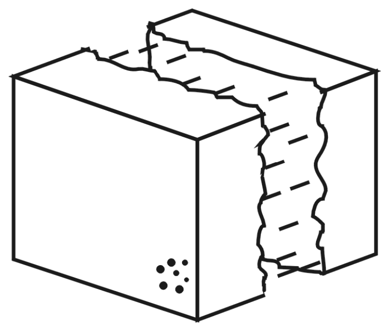
Failure of composites



(a) state of stresses

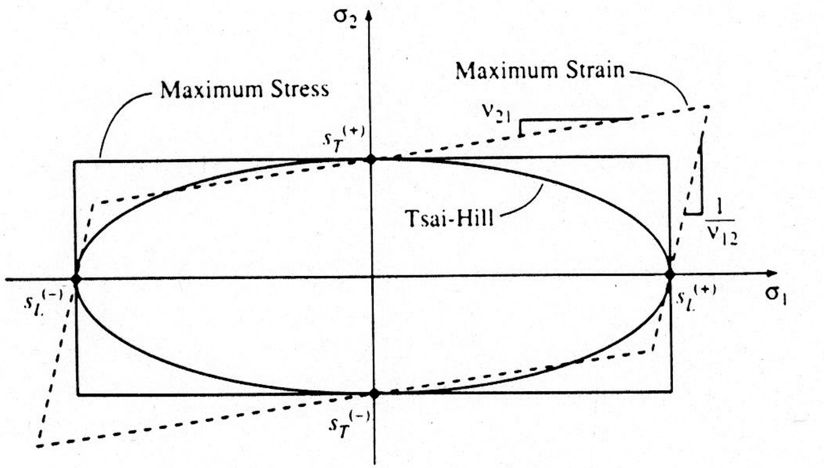


(b) matrix rupture

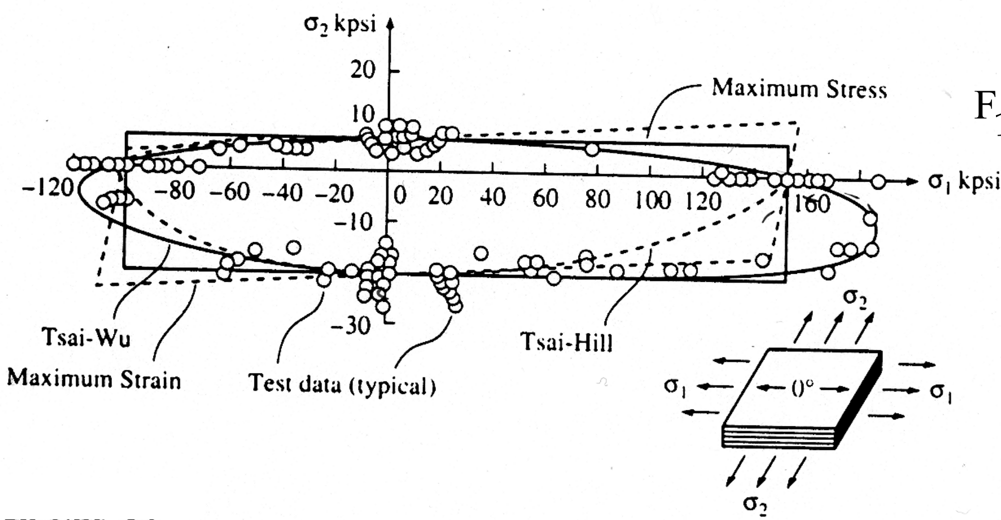


(c) decohesion, rupture of the fibes

Failure criteria



$$\frac{\sigma_1^2 - \sigma_1\sigma_2}{\left(X_1^T\right)^2} + \frac{\sigma_2^2}{\left(X_2^T\right)^2} + \frac{\tau_{12}^2}{S_6^2} = 1$$

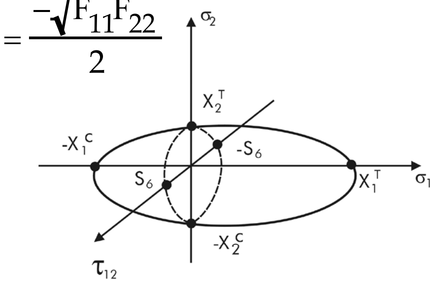


$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + 2F_{12}\sigma_1\sigma_2 = 1$$

$$F_1 = \frac{1}{X_1^T} - \frac{1}{X_1^C} \qquad F_{11} = \frac{1}{X_1^T X_1^C} \qquad F_{12} = \frac{-\sqrt{F_{11}F_{22}}}{2}$$

$$F_2 = \frac{1}{X_2^T} - \frac{1}{X_2^C} \qquad F_{22} = \frac{1}{X_2^T X_2^C}$$

$$F_{66} = \frac{1}{S_6^2}$$



First ply and final failures

Stresses in the plies

$$[\sigma]=\left[\overline{Q}\right]\left(\left\{\varepsilon^0\right\}+z\ \{\kappa\}\right)$$

1

2,3…

New stress conditions

Failure criteria

$$\frac{\sigma_1^2-\sigma_1\sigma_2}{\left(\sigma_{u,1}^T\right)^2}+\frac{\sigma_2^2}{\left(\sigma_{u,2}^T\right)^2}+\frac{\tau_{12}^2}{\tau_u^2}=1$$

Failure of 1 ply

$$\left(\overline{Q_{ij}}\right)_{\text{pli rompu}}=0$$

$$A_{ij}=\sum_{k=1}^{N}\left(\overline{Q_{ij}}\right)_k\left(z_k-z_{k-1}\right)$$

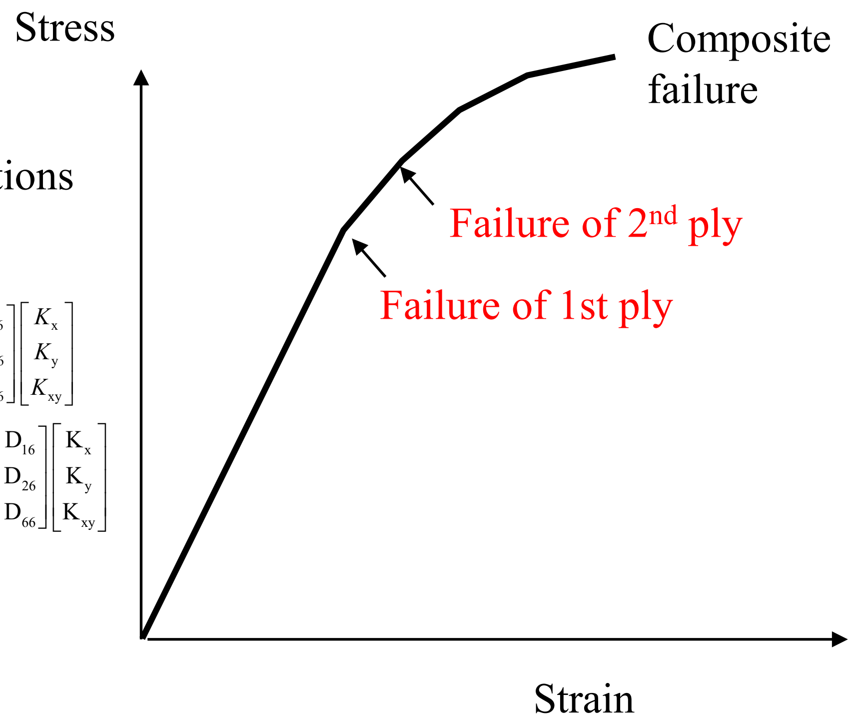
$$B_{ij}=\frac{1}{2}\sum_{k=1}^{N}\left(\overline{Q_{ij}}\right)_k\left(z_k^2-z_{k-1}^2\right)$$

$$D_{ij}=\frac{1}{3}\sum_{k=1}^{N}\left(\overline{Q_{ij}}\right)_k\left(z_k^3-z_{k-1}^3\right)$$

Re-calculation

$$\begin{bmatrix}N_x\\N_y\\N_{xy}\end{bmatrix}=\begin{bmatrix}A_{11}&A_{12}&A_{16}\\A_{12}&A_{22}&A_{26}\\A_{16}&A_{26}&A_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\\varepsilon_y^0\\\gamma_{xy}^0\end{bmatrix}+\begin{bmatrix}B_{11}&B_{12}&B_{16}\\B_{12}&B_{22}&B_{26}\\B_{16}&B_{26}&B_{66}\end{bmatrix}\begin{bmatrix}K_x\\K_y\\K_{xy}\end{bmatrix}$$

$$\begin{bmatrix}M_x\\M_y\\M_{xy}\end{bmatrix}=\begin{bmatrix}B_{11}&B_{12}&B_{16}\\B_{12}&B_{22}&B_{26}\\B_{16}&B_{26}&B_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\\varepsilon_y^0\\\gamma_{xy}^0\end{bmatrix}+\begin{bmatrix}D_{11}&D_{12}&D_{16}\\D_{12}&D_{22}&D_{26}\\D_{16}&D_{26}&D_{66}\end{bmatrix}\begin{bmatrix}K_x\\K_y\\K_{xy}\end{bmatrix}$$

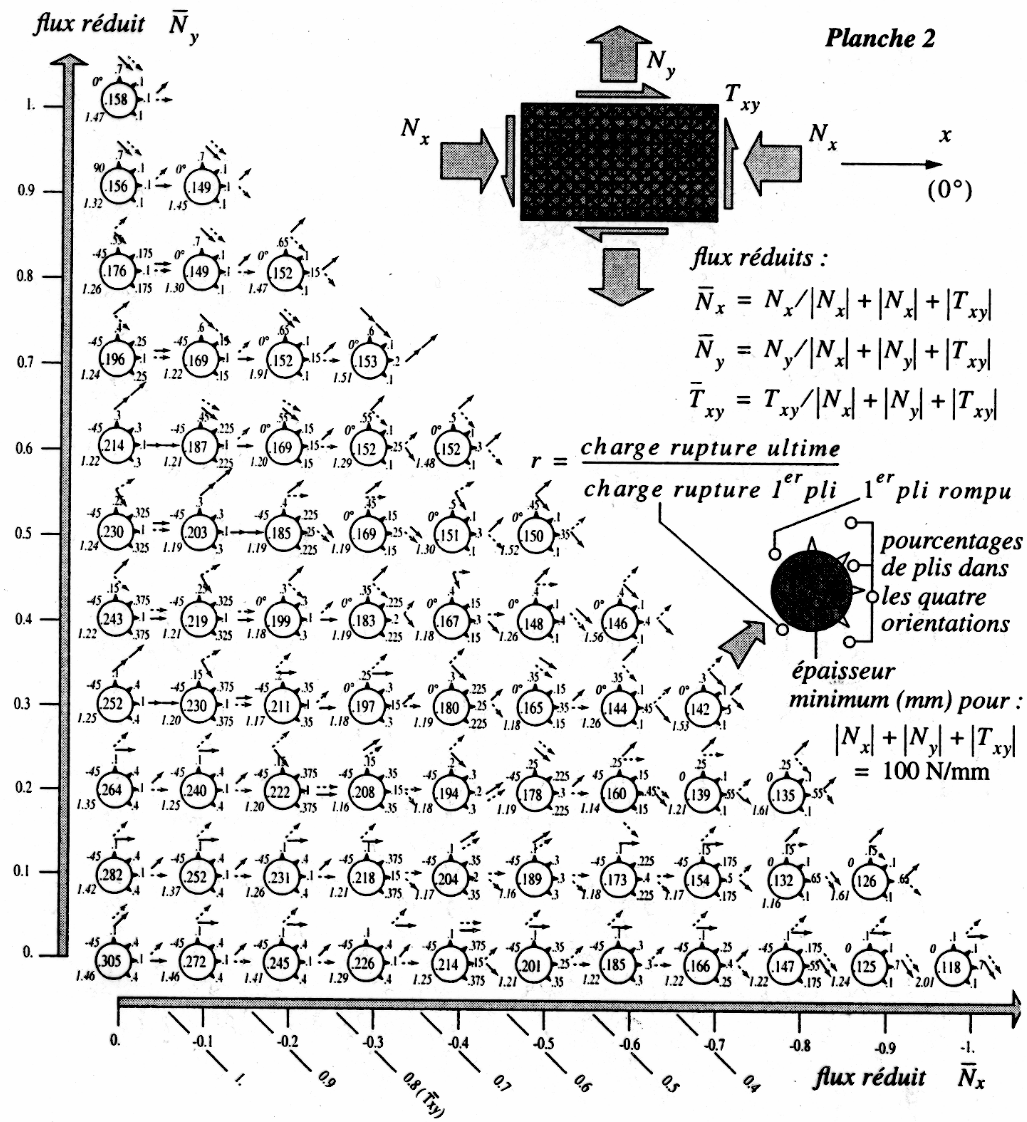


Design maps

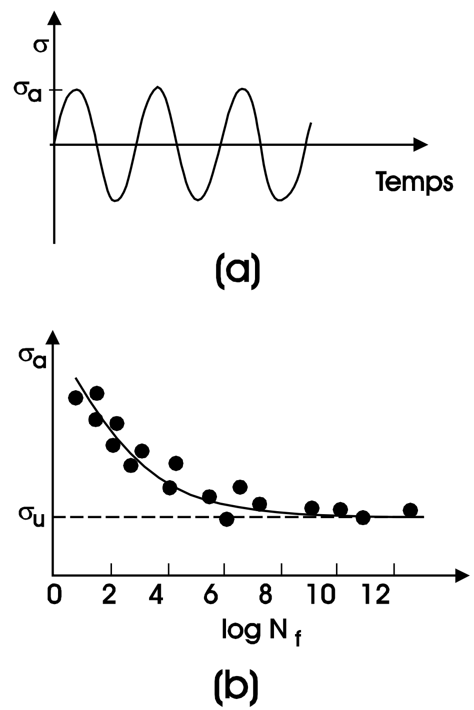
Composition optimum d'un stratifié carbone/époxyde

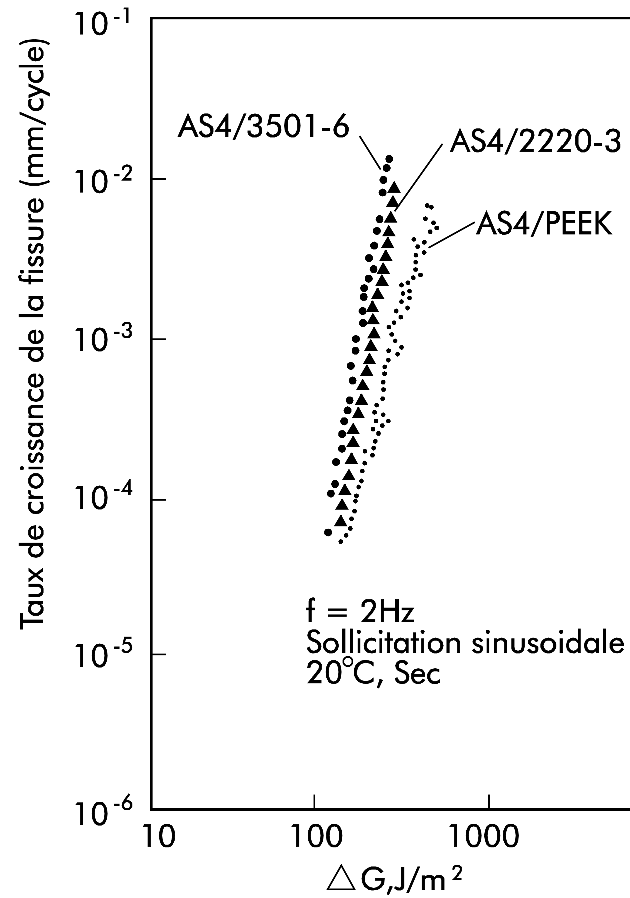
$V_f = 0{,}6$; caractéristiques du pli : *cf.* annexe 1 ou paragraphe 3.3.3.

10 % minimum de plis dans chaque direction 0°, 90°, + 45°, – 45°

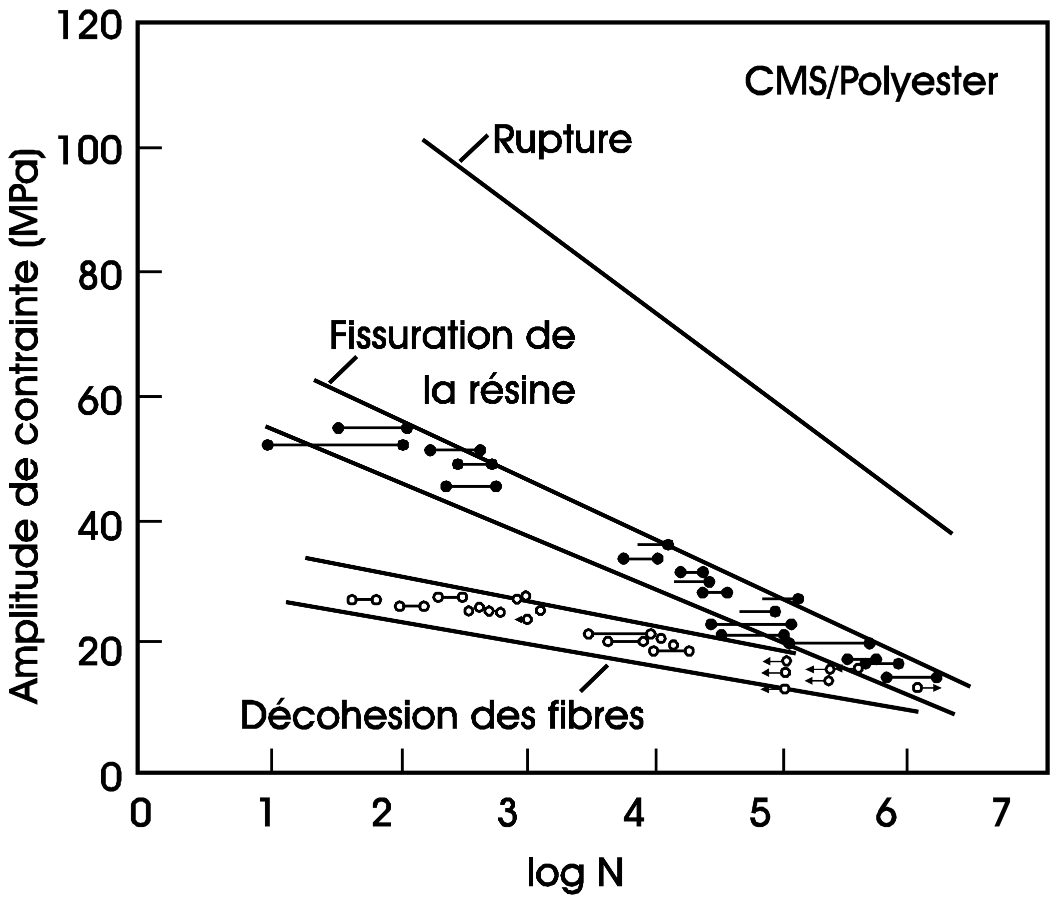


Fatigue of composites

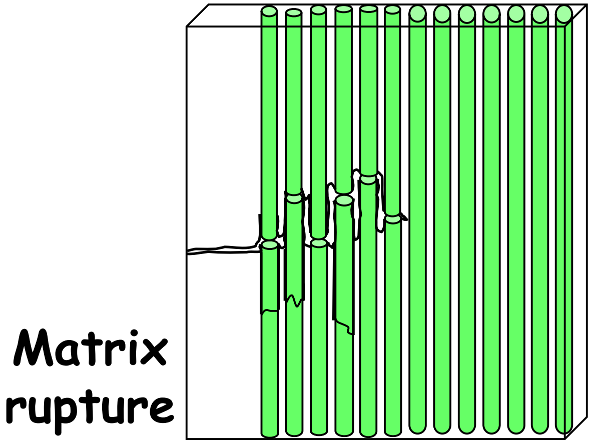


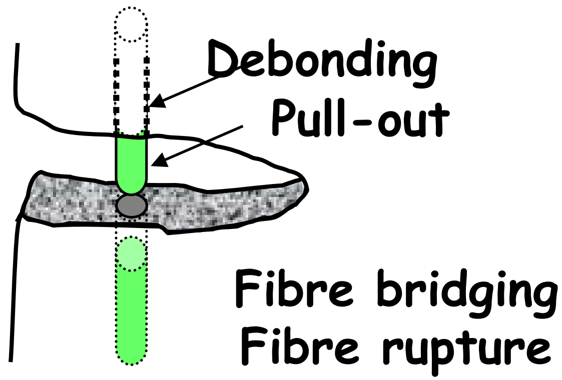


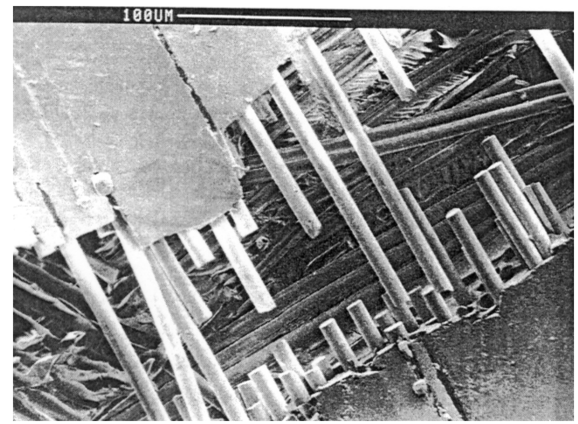
Fatigue of composites



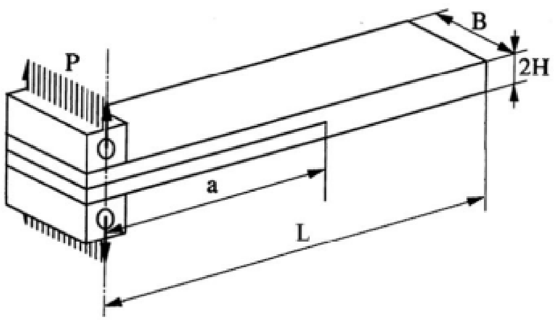
Damage in composites

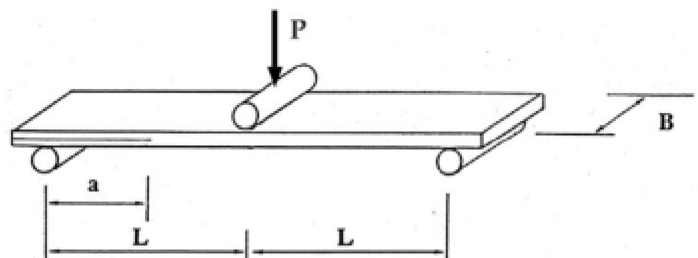




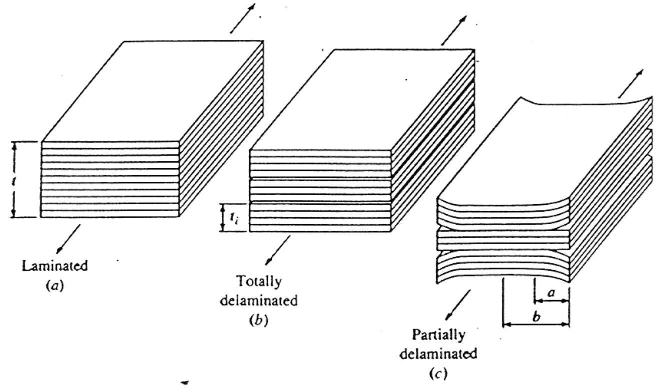


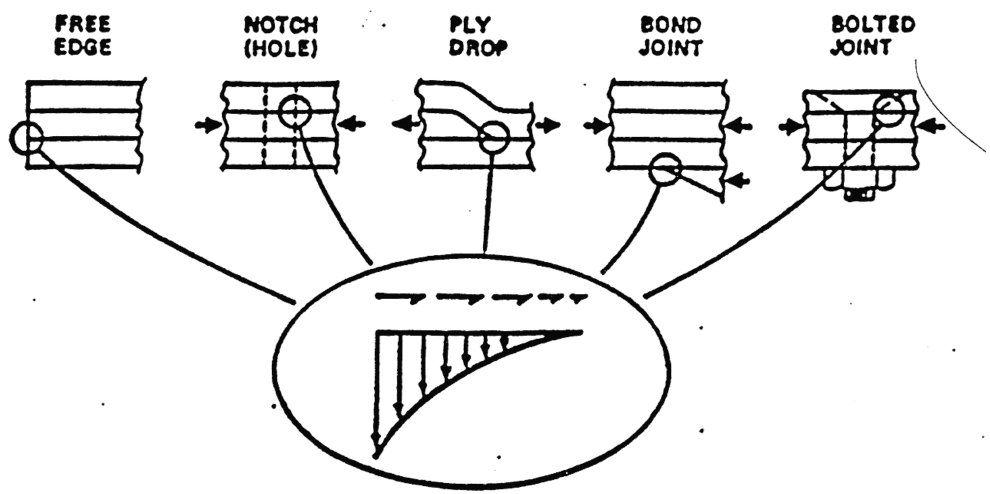
Energy release rate G, modes I, II crack propagations

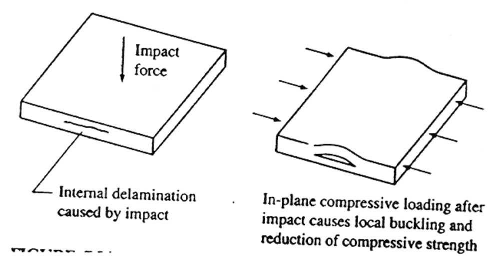




Delamination







Design

Micro and macromechanics

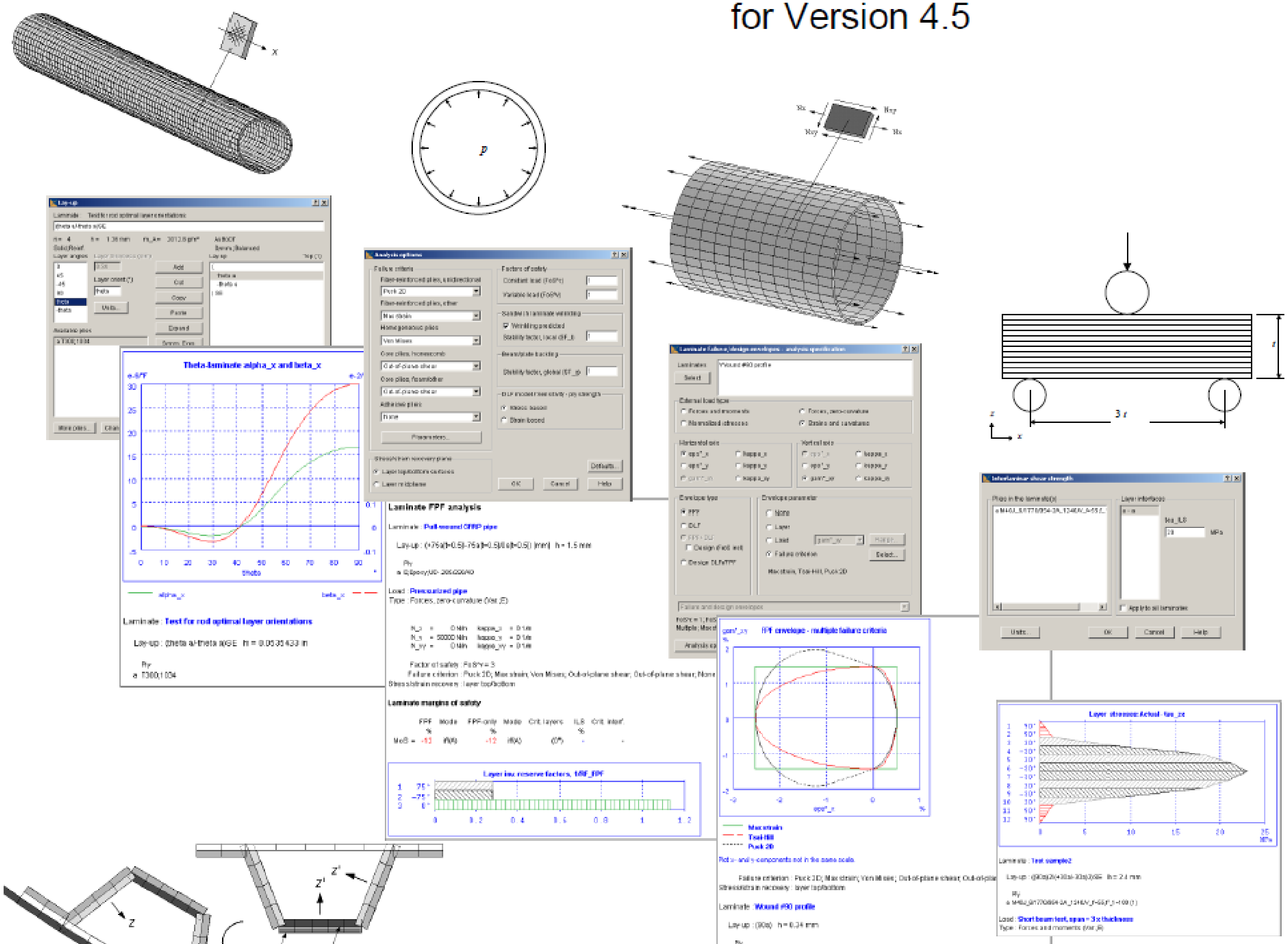
Mechanical tests

CADFEM tools

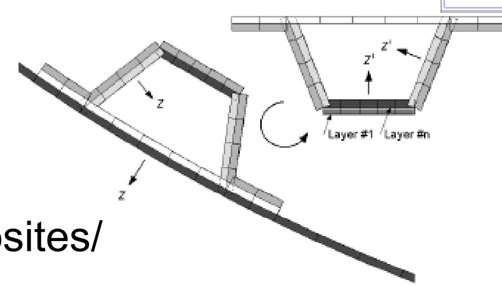
Applications

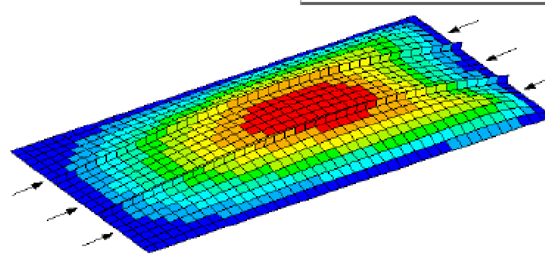
Layered composites

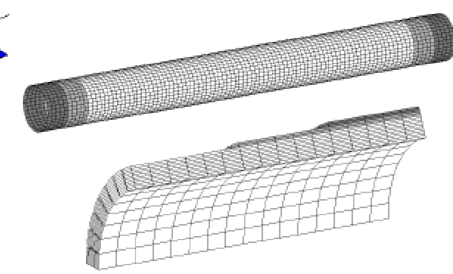
for Version 4.5



ESAComp
Example Cases



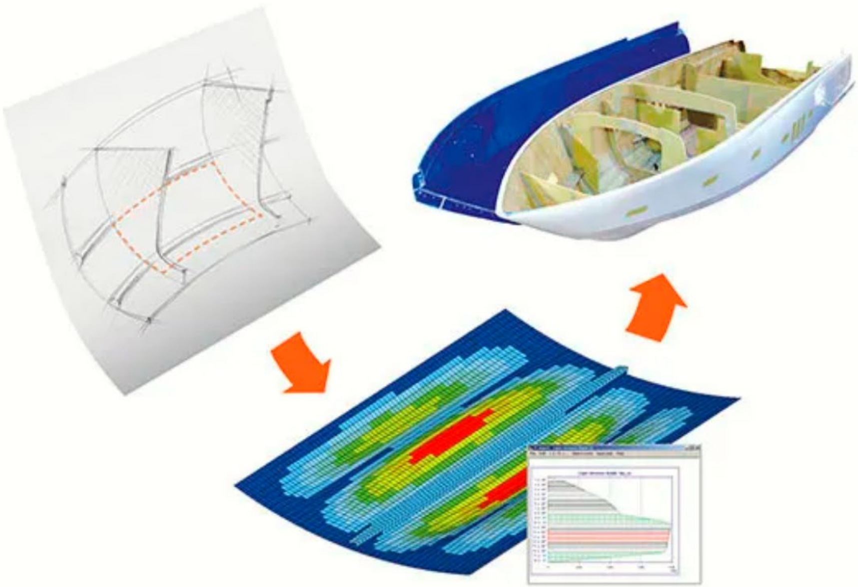


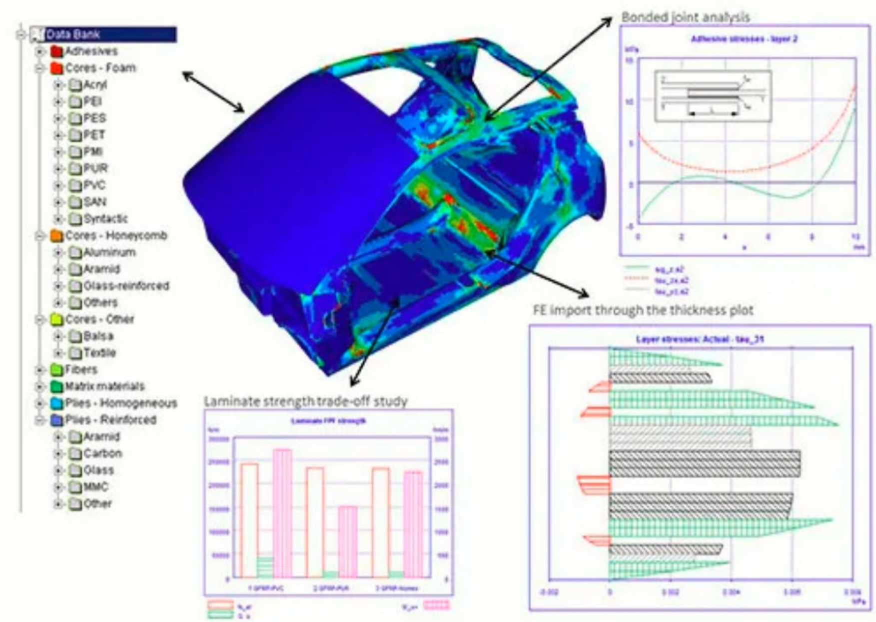


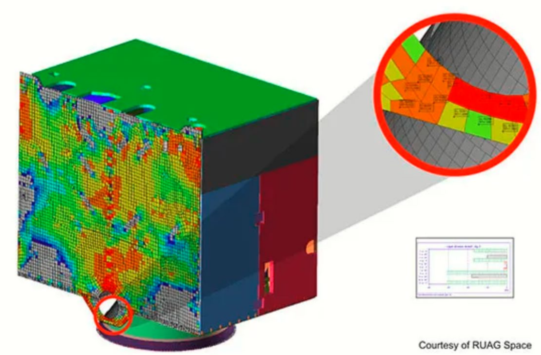
https://www.altair.com/composites/

vdi.epfl.ch STI-WINDOWS 11 Altair ESAComp

FEM







https://filecr.com/windows/esacomp/?id=13436849765

Multiscale analysis

Example: Prof. Sung Kyu Ha, Hanyang University

Micro-meso-macro scale
Impact simulation
Damage in thin and thick laminates
Fatigue life prediction
Blade design

More to come

Textile composites

Exercises on Moodle

Project: your team, your topics

https://docs.google.com/spreadsheets/d/1rOpo4XUvnsMO0ddnnhDwqJoY_zVaBDXZi_EYL0t1tRg/edit?gid=0#gid

isa 180925

71/71	Name First Name	Sciper	Section	Teams		Names
1	Artru Thomas	329649	MX	Team 1	1	Filippo Andrea Saverio
2	Baiada Elias Julien	361290	MX		2	Greulich Eric
3	Berchtold Ludovic	363629	MX		3	Bréas Davide Raphaël
4	Bertrand Alexis Walter	345322	MX		4	Cazzulani Federico
5	Bisetti Lucas Dino	345160	GM		5	Pierre-Arnaud Vals
6	Bréas Davide Raphaël	344476	MX		6	Pin-Ying Chen
7	Breugnot Tom Didier Yves	361538	MX	Team 2	7	Ludovic Berchtold
8	Carandang Gian-Angelo	296735	GM		8	Diane Meslin
9	Cauvin Marie Anna Martha Jean	346001	GM		9	Roman Rechsteiner
10	Cavoret Martin Noël Claude	329148	GM		10	Sarah Naili
11	Cazzulani Federico	386924	GM		11	Loris Dubi
12	Chalhoub Marc-Antonio	340428	GM		12	Thomas Sancenot
13	Chen Ko-Yu	419062	CGC_ECH	Team 3	13	Vivek Agasthya
14	Chen Pin-Ying	395229	MX		14	Yasar Sude
15	Cuvillon Rose Alice	361770	MX		15	Lee Jeanne
16	Debrot Nicolas	329951	GM		16	Nagy Orsi
17	Douarche Victoria Alizée	361916	MX		17	Orkun Özyurt
18	Dubi Loris	362613	MX		18	Vincente Galdini
19	Ducommun Léandre	326851	MX	Team 4	19	Martin Kelles
20	Dupille Charles Rémy Marie Max	347080	GM		20	Hugo Subtil
21	Eggen Nathan	347058	GM		21	Nicolas Debrot
22	Filippo Andrea Saverio	341036	MX		22	Romain Lattion
23	Fotius Antoine Georges Henri	394196	MX		23	Frederik Kaiser
24	Freytag Julius Gideon John	417744	MX_ECH		24	
25	Galdini Vincente Annibale	302002	GM	Team 5	25	Rose Cuvillon
26	Givry Gabriel	341068	GM		26	Martin Cavoret
27	Gnamus Jan	388105	ING_PHYS		27	Thomas Menadi
28	Goutal Lea Soraya	406443	MX		28	Victoria Douarche
29	Gouttenoire Quentin Christian Je	346918	GM		29	Louise Holland
30	Greffe Florent Xavier Marie	344833	GM		30	Vincent Vos
31	Greulich Eric	346411	MX	Team 6	31	Petrov Viktor
32	Grillet-Aubert Rémy Thibault Jos	361138	MX		32	Givry Gabriel
33	Holland Louise Hélène	415566	GM_ECH		33	
34	Ivanoff Valentine Léonor	345249	MX		34	
35	Joncquel Hugo Etienne	326689	MX		35	
36	Kelles Martin Jacques Jean	312294	GM		36	
37	Kieffer Camille Marie	340653	MX	Team 7	37	Daniele Palombi
38	Kruzic Natacha Helena Victoria	325854	MX		38	Lea Goutal
39	Kühni Cyran Olivier	341623	GM		39	Alessandro Mito
40	Kuntze Thomas Roland Pierre	363387	GM		40	Alexis Bertrand
41	Lasserre Emilie Charlotte	362484	MX		41	Elias Baïada
42	Lattion Romain	347395	GM		42	Rémy Grillet-Aubert

Team work
Materials
Design
Value proposition
Manufacturing
Cost
Sustainability
Report
Presentation

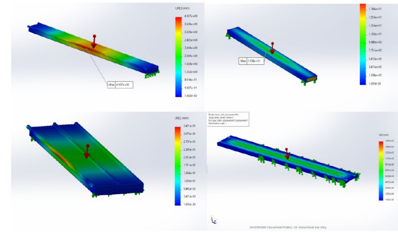


FIGURE 7 – Examples of structural simulations done during the iterative process to design the roof panels

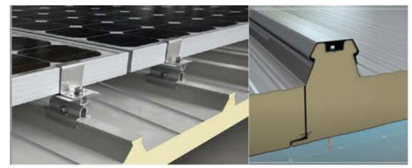
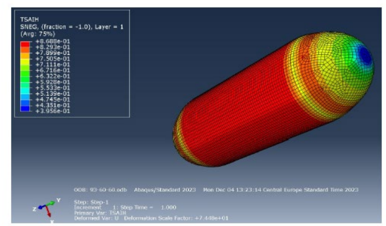
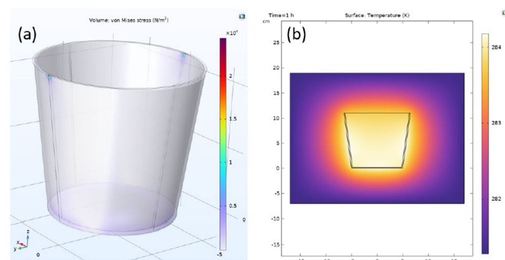


FIGURE 8 – Example of a solar panel attachment system on an insulated roof panel existing on the market



(c) (90/60/-60) orientation



Roof panels
Oxygen bottles
e-scooter
Biodegradable flower pots
Battery tray
Chicken feathers
composite longboard
Artificial skins
Vegan microfiber balls
.....

Course outline 2025

09.09.2025	Introduction to course and learning objectives, group project, constituents	peb/vm
16.09.2025	Composites Applications and background on composite processing	vm/mw
23.09.2025	Background on composite mechanics	peb
30.09.2025	Sandwich structures and textile composites	peb
07.10.2025	Structural design/joining with composites	cb
14.10.2025	Projects follow-up session I	peb/vm/mw
21.10.2025		vacation
28.10.2025	Smart composites	vm
04.11.2025	Composites for biomed and sport	peb
11.11.2025	Cost modelling - a tool for sustainable innovation	mw/vm
18.11.2025	Towards sustainable composites	mw
25.11.2025	Projects follow-up session II	peb/vm/mw
02.12.2025	Student project week	
09.12.2025	Student project presentations	peb/vm/mw
16.12.2025	Student project presentations and exam	peb/vm/mw